TURBULENT MASS TRANSFER AND DIFFUSION IN POLYMER SOLUTIONS REDUCING FRICTION DRAG

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A comparison is presented between the computed and experimental data on turbulent mass transfer and diffusion in the near-wall domain of a channel and plate.

In connection with the broad application of the fluids in which a drag reduction effect appears, the problem of turbulent diffusion and mass transfer together with heat transfer is of interest. The few semiempirical methods of computing the mass and heat transfer do not, as a rule, take account of the turbulence damping singularities near the wall. The Van Driest formula for the mixing path [1-3] and the Sess formula for a turbulent fluid [4] are here most widespread. The intensity of turbulent mass and heat transfer can be computed without using comparatively complex layered schemes [5, 6] if the law of turbulent diffusion damping near the wall is known

$$D_\text{т}(y) = by^n, \tag{1}$$

where b, n are positive empirical constants, where a value of 3 or 4 is ordinarily taken for n. The question of the exact magnitude of n is for discussion. The determination of n in Newtonian fluid is based on measurements of the steady values of the substance- or heat-transfer coefficients for high Schmidt (Prandtl) values. For this, the quantity n is found from the dependence

$$\text{St} \sim \text{Sc}^{\frac{n-1}{n}}, \tag{2}$$

by variating values of the Pr or Sc numbers in a broad range, where $\text{St} = K/u_*$, K is the mass-transfer coefficient, and u_* is the dynamic velocity, $u_* = \sqrt{\tau_w/\rho}$.

In non-Newtonian fluids, polymer solutions and suspensions, the change in a broad range of Sc and Pr numbers (as is possible mainly because of the rise in consistency) under conservation of the conditions for turbulent flow development is quite problematical. Consequently, attempts were made to develop a method of determining the quantity n for Sc = const by analyzing I(t) — the transition characteristics of the diffusion current in the developed turbulent mass-transfer mode [7, 8] when using the electrochemical method. The corresponding boundary-value problem here has the form

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial y}\left\{[D + D_\text{т}(y)]\frac{\partial C}{\partial y}\right\} \tag{3}$$

with the boundary conditions

$$\begin{gathered} t = 0,\ C = C_\infty,\ y = 0; \\ t > 0,\ C_w = 0,\ y = 0; \\ t > 0,\ C = C_\infty,\ y \to \infty. \end{gathered} \tag{4}$$

Underlying the method of determining n are approximate analytic and numerical solutions of (3).

Let us examine the results of investigating the nonstationary mass transfer. Analysis of the nonstationary mass-transfer equations (3) shows that a change in the law of turbulent diffusion damping (1) should affect the transition characteristic of the diffusion current I(t), including the build-up time of the stationary mass transfer. Our tests showed that

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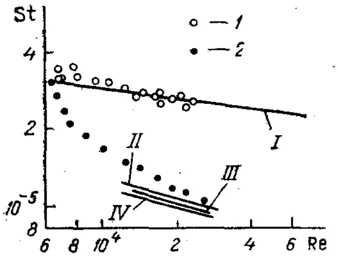

Fig. 1

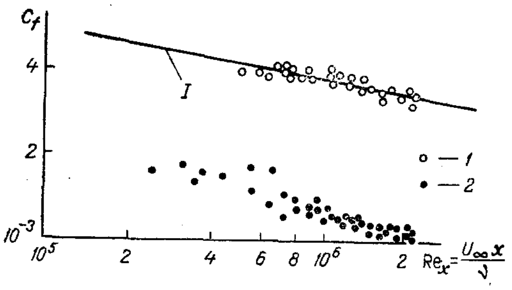

Fig. 2

Fig. 1. Mass transfer of channel walls during turbulent flow of PEO WSR-301 solutions: 1, 2) experimental results, $C_p = 0$ and 0.01%; I) computation using (11); II) computation using (8) and (9); III) approximation of test data [13]; IV) data of a computation [12].

Fig. 2. Local friction of a plate: I is the theory for the turbulent mode [15]; 1) measurements in an electrolyte solution without PEO admixture; 2) in an electrolyte solution with PEO admixtures, $C = 0.007\%$.

admixtures of the polyethylene oxide (PEO) WSR-301 result in a substantial increase in the time of mass-transfer stabilization [9]. In conformity with the method developed in [7] for the determination of the exponent in the turbulent diffusion damping power law (1), the value of n for developed turbulent flow turned out to be 4.02 ± 0.11 for a solute and 3.00 ± 0.07 for the PEO WSR-301 solution $0.002\% < C_p < 0.03\%$, as a result of processing 59 transition characteristics.

By using the data of steady-state and nonstationary turbulent mass transfer, the expression for the limit law of turbulent diffusion damping in polymer solutions of the PEO WSR-301 type has the form

$$\frac{D_\text{т}}{\nu} = 1.6 \cdot 10^{-4} y_+^3. \tag{5}$$

For a pure solute our test data are in good agreement with the T. S. Kornienko formula [10]

$$\frac{D_\text{т}}{\nu} = 2.7 \cdot 10^{-4} y_+^4. \tag{6}$$

On the basis of (5) an expression can be derived for computation of the mass-transfer coefficient in solutions reducing the drag

$$\text{St} = 0.045\text{Sc}^{-2/3} \sqrt{\frac{\lambda}{8}}. \tag{7}$$

Using the Virk limiting asymptote ($\lambda = 0.21\ \text{Re}^{-0.55}$), it is possible to obtain from (7) [9]

$$\text{St} = 0.021\text{Re}^{-0.275}\text{Sc}^{-2/3}. \tag{8}$$

The authors of [11, 12] used the relationship for the turbulent diffusion coefficient from (5) and obtain an expression to compute the turbulent mass transfer in a channel under maximal drag reduction conditions

$$\text{St} = 0.022\text{Re}^{-0.28}\text{Sc}^{-2/3}, \tag{9}$$

$$\text{St} = 0.0167\text{Re}^{-0.275}\text{Sc}^{-2/3}. \tag{10}$$

Results of electrochemical measurements of the mass transfer on a 1 × 1-cm channel wall are represented in Fig. 1 where limiting asymptotes of mass-transfer reduction from [13] are additionally superposed as are also those obtained in [9, 11, 12] on the basis of using the turbulent diffusion damping law according to (5). As is seen from Fig. 1, the agreement between the test data (dark points) and the computations (lines II, III, IV) is satisfactory under maximal drag reduction conditions. In the absence of polymer admixtures the test data are approximated by the S. S. Kutateladze formula [14]

$$\text{St}\sqrt{\frac{8}{\lambda}} = 0.115\text{Sc}^{-3/4}, \tag{11}$$

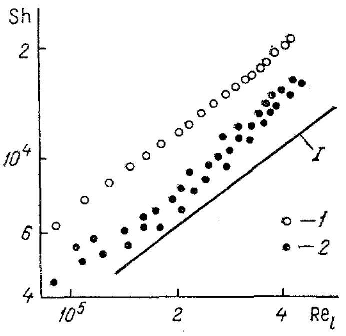

Fig. 3

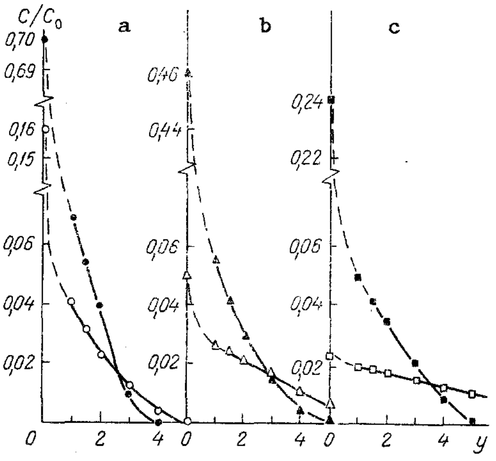

Fig. 4

Fig. 3. Mass transfer of a continuously streamlined plate: 1) $C_p = 0$; 2) 0.007%; I) computation using (17).

Fig. 4. Concentration profiles of the depolarizer injected in the boundary layer for x = 36 (a), 68 (b), and 168 mm (c) distances from the slot: open points are measurements in an electrolyte, and dark points in the PEO solution (C_p = 0.1%). C/C_0, %; y, mm.

where $\lambda = 0.316/\mathrm{Re}^{0.25}$.

Let us examine the possibility of using the law of turbulent diffusion damping according to (5) to compute the turbulent mass transfer of a plate. The mass transport differential equation for this case has the form

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left[(D+D_{\text{т}})\frac{\partial C}{\partial y}\right] \tag{12}$$

with the boundary conditions

$$\begin{gathered}C=C_w,\ x>0,\ y=0;\\ C=C_\infty,\ x>0,\ y\to\infty;\\ C=C_\infty,\ x=0,\ y>0.\end{gathered} \tag{13}$$

The corresponding heat-transfer problem was examined in [15]. The Spaulding method [15] for the solution of this mass-transfer problem in a Newtonian fluid under the condition Sc ≫ 1 and the turbulent diffusion damping law in the form (6) was used in [10]. Using the Mises variables, introducing the stream function, making everything dimensionless and effecting certain other transformations, (12) can be reduced to the form

$$y_+\frac{\partial C_+}{\partial x_+}=\frac{\partial}{\partial y_+}\left[\left(\frac{1}{\mathrm{Sc}}+by_+^n\right)\frac{\partial C_+}{\partial y_+}\right] \tag{14}$$

with the boundary conditions

$$\begin{gathered}C_+=1,\ x_+>0,\ y_+=0;\\ C_+=0,\ x_+>0,\ y_+\to\infty;\\ C_+=0,\ x_+=0,\ y_+>0.\end{gathered} \tag{15}$$

Here

$$y_+=\frac{yu_*}{\nu};\ x_+=\int\limits_0^x\frac{u_*}{\nu}dx;\ C_+=\frac{C-C_\infty}{C_w-C_\infty}. \tag{16}$$

The dependence $K/u_* = f(x_+)$, which can be found from an approximation of the numerical solution of (14) for specific values of n must be known for an analytic solution of (14). Such an approach is used in [10] for n = 4 by using graphs of the numerical analysis by Son and Hanratty [16]. An analogous procedure can also be realized for our case, i.e., for the damping law in the form (5). In sum, the expression to compute the number Sh will have the following form

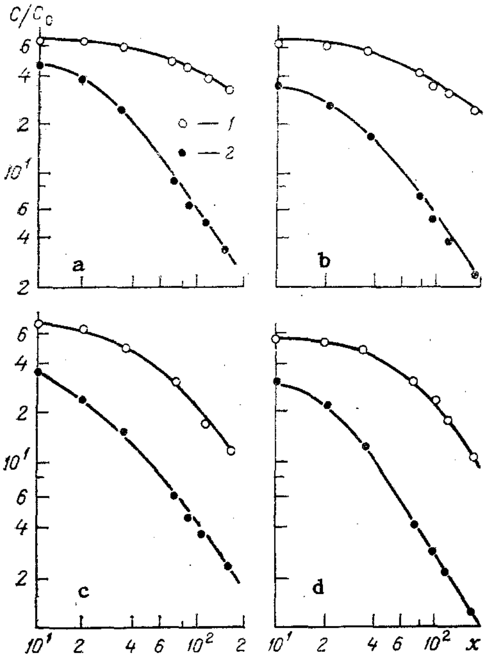

Fig. 5. Concentration distribution along a streamlined wall: 1) $C_p = 0$; 2) 0.1%. x, mm.

$$\overline{\mathrm{Sh}} = \frac{\overline{K}L}{D} = B\mathrm{Sc}^{1/3}\,\frac{N^{1/2}\mathrm{Re}_L^{1-\frac{m}{2}}}{1-\dfrac{m}{2}}\,G(\xi), \tag{17}$$

where $\xi = b^{3/n}\mathrm{Sc}^{3/n-1}x_+$; N and m are the exponent and numerical coefficient in the dependence

$$\frac{C_f}{2} = N\,\mathrm{Re}_x^{-m}. \tag{18}$$

Values of the quantities B and $G(\xi)$ are found from the approximation of the numerical solution of (14), from which it follows that $\xi > 1$ for $\mathrm{Sc} > 10^3$ and $\mathrm{Re} > 2\cdot 10^5$, and respectively, $G(\xi) \approx 1$, $B = 0.9\ b^{1/n}$.

A turbulent flow was produced near the plate surface in our tests by installing it in a fluid-filled rotating annular channel of 542-mm diameter and 125-mm width of "squirrel cage" type. A 98-mm-long and 1-mm-wide plate intended to measure the friction was fabricated from strong glass-plastic with rectangular 1 × 0.06 mm electrodes embedded and carefully polished flush with the surface at different distances from the leading edge. The integral mass transfer of the plate was determined by using platinum foil $x_L = 42$ mm long glued thereon. As in the previous case solutions of electrolytes based on $2.5\cdot 10^{-2}$ kmol/m³ of $K_3[Fe(CN)_6]/K_4[Fe(CN)_6]$ and $2.5\cdot 10^{-1}$ kmol/m³ of K_2SO_4 were used to measure the local friction values and the integral mass transfer while ferrocyanides were the working depolarizer. A correlation method of processing the oscillograms of the diffusion current fluctuations of two microelectrodes arranged near the leading edge was used to measure the velocity U_∞ at the spreading point which is strictly speaking not equal to the velocity of quasisolid rotation. Represented in Fig. 2 are results of measuring the local drag coefficients which are in good agreement with the data in [15, 17] for a solute without PEO admixtures. The polyox WSR-301 admixture in a C = 0.007% concentration resulted in a substantial reduction in the values of C_f (up to 68%) in the developed turbulent flow section. Test data referred to an electrolyte solution without and with PEO admixtures are approximated by the respective expressions

$$\frac{C_f}{2} = 0.0296\mathrm{Re}_x^{-0.2}, \tag{19}$$

$$\frac{C_f}{2} = 0.24\mathrm{Re}_x^{-0.41}. \tag{20}$$

The measurement results as well as the mass-transfer computation using (17) and the relation (20) are represented in Fig. 3. The lower computed values of Sh as compared with the experimental data are due to the fact that the developed turbulent flow is not realized on the whole length of the plate as is assumed in the computation.

A reduction in the turbulent diffusion intensity in polymer solutions is recorded not only near the wall. The authors of [18] executed a series of experiments in which the radial dispersion of a fluid dye inserted on the tube axis in a pure water or weak polymer solution flow was measured. The measurements showed that the transverse turbulent mixing intensity in the polymer solution is lower than in water. Qualitatively the same results are obtained in [19] on the basis of an analysis of diffusion wakes behind sources located at different points of a channel of 4.4 × 4.4 cm section and in [20] during the ejection of a PEO solution and water through an annular slit, and also in [21, 22].

When studying impurity propagation from a slot source directed at a small angle to the channel wall, the selection of the method of determining the concentration in direct proximity to the wall, at distances of $y_+ < 100$, should be especially carefully noted. The holes in the wall utilized to select the fluid samples induce perturbations. It can consequently be considered that the sample selection is from a layer of thickness equal to the hole diameter in this case.

The electrochemical method of measuring the impurity distribution [23] permits information to be obtained about the magnitude of the concentration at distances from the wall that do not exceed the dimensions of the diffusion boundary layer of the measuring electrode. Practically no perturbations are here induced and the diffusion layer thicknesses can be sufficiently small without exceeding the thickness of the viscous sublayer $y_* \sim 5$.

The apparatus for impurity injection into the channel boundary layer in our tests was a slot with transverse dimension h = 0.4 mm directed at a ∿ 7° angle to the wall. Microelectrodes superposed on the glass insert of the channel by an electrovacuum deposition method were used as concentration meters in the near-wall domain. The measurements at the flow core were performed by miniature wedge electrochemical sensors. The impurity mass flow rate was chosen in such a manner that the influence of the fluid flow out of the slot on the velocity field was eliminated in practice.

The measured "passive" impurity concentration profiles (ferrocyanides) are represented in Fig. 4 for $\mathrm{Re} = 7.5 \cdot 10^4$. As is seen from the figure, if values of the concentration on the wall are taken into account, which are substantially higher than at a 1 mm distance from the wall, then the concentration profile has a noticeable inflection. Such a phenomenon is not noticed if the concentration at the wall is determined by using a drainage hole. As tests with glycerine solution injections showed, an increase in impurity concentration at the wall is not related to an increase in viscosity. This can be explained by the fact that the turbulent diffusion coefficient is quite small (see (5)) in a domain deeply submerged in the viscous sublayer ($y_+ = 0.5 - 1$). The concentration profile in the core of the flow ($y_+ > 100$), constructed in the coordinates $C/C_m = f(y/y_{1/2})$ (not presented in the figure), extends the test data well for different values and agrees with the results of experiments known from [24, 25].

Represented in Fig. 4 are also concentration profiles of a solution of ferrocyanides with the polymer WSR-301 injected into the boundary layer. The flow rate through the slot and the initial ferrocyanide concentration were identical in both cases. Admixture of the polymer (C = 0.1%) resulted in an abrupt rise in the display impurity concentration in direct proximity to the wall and in a noticeable narrowing of the impurity propagation zone in the transverse direction. Measurement of the impurity concentration in the longitudinal direction for y ≈ 0 showed that the insertion of the PEO in a flow retards the tempo of the concentration reduction along the flow. The impurity concentration distribution in the near-wall domain can be expressed in the form of an empirical dependence

$$\frac{C}{C_0} = A\frac{q}{D_\text{т} x}\left(\frac{x}{h}\right)^{-p}, \tag{21}$$

where A and p depend on the fluid properties.

From the viewpoint of an increase in the relative impurity concentration $\bar{C} = C/C_0$ near the wall the polymer efficiency can be estimated by using (21), (5), and (6). Thus for fixed values,

$$\frac{\bar{C}_\text{p}}{\bar{C}} = \frac{D_\text{т}}{D_{\text{т}\text{p}}} = \frac{2.7 \cdot 10^{-4} y_+^4}{1.6 \cdot 10^{-4} y_+^3} = 1.69 y_+. \tag{22}$$

Setting $y_+ = 5$ we obtain $\bar{C}_p/\bar{C} = 8.4$, which is in good agreement with test data (Fig. 5) on the portion of the turbulent-diffusion stabilizing process (in Fig. 5c and d for x > 40 mm). Our test data are approximated in the stabilizing-diffusion domain by the expression

$$\frac{C}{C_0} = \frac{1.2 \cdot 10^{-3} q}{D_{\text{T}} x} \left(\frac{x}{h} \right)^{-0,2}. \tag{23}$$

As the concentration and mass flow rate of the polymer solution increases, the tempo of the concentration diminution along the wall is lowered (Figs. 5a and b). For this case the values of the quantities A and p in (21) will be different.

In conclusion, let us note that the good agreement of the computational and experimental data for the cases considered above affords a foundation for using a power-law of turbulent diffusion damping with exponent n = 3 for other turbulent mass-transfer conditions in polymer solutions that reduce their friction drag.

NOTATION

C, C_m, C_w, C_∞, C_0, concentration of the working depolarizer, its maximal value, the value at the wall, in the volume of the electrolyte solution, at the exit from the slot source, $\text{kmole} \cdot \text{m}^{-3}$; C_p, polymer concentration, %; $\bar{C}$, $\bar{C}_p$, relative concentrations of depolarizer without and with PEO admixtures, respectively; D, depolarizer diffusion coefficient, $\text{m}^2 \cdot \text{sec}^{-1}$; D_T, D_{Tp}, turbulent diffusion coefficients for the electrolyte and the polymer solutions; I, limiting diffusion current, $\text{A} \cdot \text{m}^{-2}$; q, fluid flow rate, $\text{m}^3 \cdot \text{sec}^{-1}$; L, characteristic dimension, the plate length; m; x_L, length of the plate mass-transfer surface, m; τ_w, friction stress, $\text{N} \cdot \text{m}^{-2}$; $C_f = 2\tau_w/\rho U_\infty^2$, plate drag coefficient; $\lambda = 2\Delta p/\rho U_\infty^2 \, d/L$, drag coefficient in the tube; Δp, pressure drop in the length L; U_∞, flow velocity, $\text{m} \cdot \text{sec}^{-1}$; x, y, coordinates; t, time, sec; ν, kinematic viscosity, $\text{m}^2 \cdot \text{sec}^{-1}$; u, v, velocity components; $Sc = \nu/D$, Schmidt number; $Re_x = U_\infty x/\nu$, $Re = U_\infty L/\nu$, Reynolds number; $Pr = \nu/a$, Prandtl number; and a, thermal diffusivity coefficient $\text{m}^2 \cdot \text{sec}^{-1}$.

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VISCOUS DISSIPATION IN JETS

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On the basis of the boundary-layer equations we consider self-similar flow regimes of a jet with an exponential dependence of the viscosity on temperature.

We consider an incompressible fluid jet ejected from a circular nozzle with an initial temperature not equal to the temperature of the surrounding medium. It is assumed that heat is produced in the flow because of viscous dissipation, and that the temperature dependence of the viscosity is given by

$$\mu = \mu_0 \exp(E/RT). \tag{1}$$

This relation is valid for the viscosities of condensed media in the temperature region of practical interest.

It was observed in [1] that for a given pressure drop there was a sharp change in the thermal behavior of the flow of a Newtonian fluid in an infinite pipe for $\chi > \chi_{cr}$, if the temperature dependence of the viscosity is given by (1). This was called a hydrodynamic thermal explosion.

In the present paper we study the possibility of a similar effect in an incompressible fluid jet.

The equations of motion and the heat equation in the boundary-layer approximation for the flow of the jet have the form [2]:

$$\begin{gathered} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(\nu y\frac{\partial u}{\partial y}\right), \\ u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{a}{y}\frac{\partial}{\partial y}\left(y\frac{\partial T}{\partial y}\right) + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \\ \frac{\partial}{\partial x}(yu) + \frac{\partial}{\partial y}(yv) = 0. \end{gathered} \tag{2}$$

With the boundary conditions:

$$\frac{\partial u}{\partial y} = 0, \frac{\partial T}{\partial y} = 0,\ v = 0,\ y = 0;\ T \to T_\infty,\ u \to 0,\ y \to \infty. \tag{3}$$

In the self-similar formulation of the problem the initial conditions are replaced by the integral relations

$$I = I_0 = \pi\rho u_0^2 d^2/4,\ Q = Q_0 = \pi\rho c_p u_0 T_0 d^2/4. \tag{4}$$

The subscript zero indicates that the corresponding quantity is evaluated at the nozzle cross section.

We apply the method of local similarity [3]:

$$u = u_m(x) f'(\varphi)/\varphi,\ \Delta T = \Delta T_m(x)\,\theta(\varphi),\ \varphi = y/\delta(x). \tag{5}$$

Using the Frank–Kamenetskii expansion [4], we transform the system of equations (2) to the following form, using the method of local similarity:

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